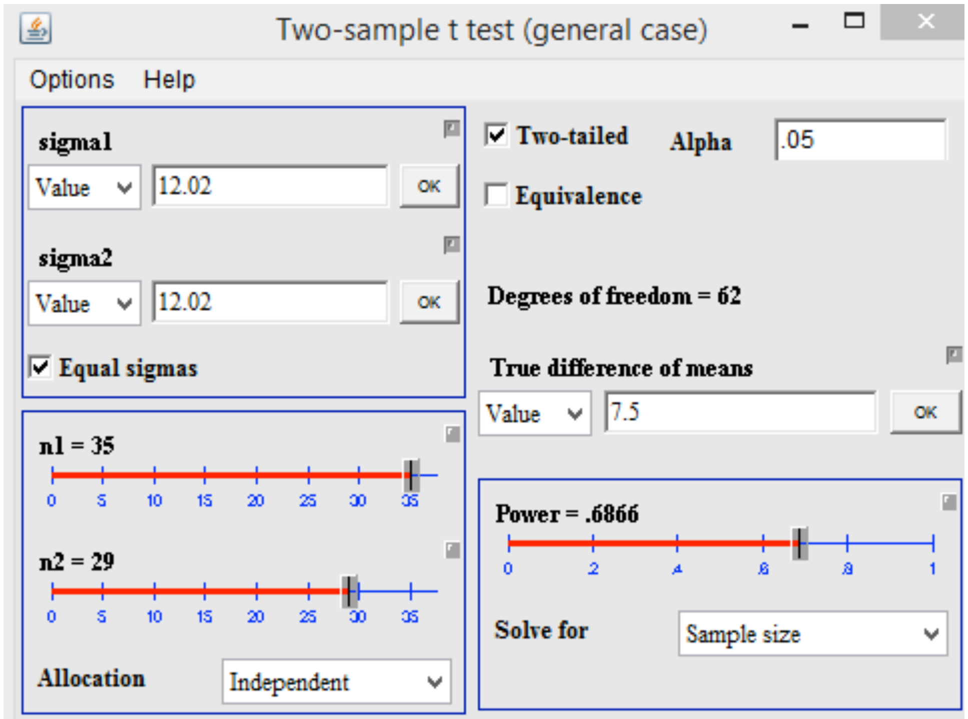
Chapter 15—Power

15.1 Power using Lenth's program.



15.3 Effect size and power for socially desirable responses:

Assume the population mean = 4.39 and the population standard deviation = 2.61

a) Effect size:

$$\gamma = \frac{\mu_1 - \mu_0}{\sigma} = \frac{4.39 - 3.87}{2.61} = \frac{0.52}{2.61} = .20$$

b) delta:

$$\delta = .20\sqrt{36} = 1.20$$

c) power = .22 for a one-sided test

Notice that the value of δ here is exactly the same as the value of *t* in that example. This is as it should be.

15.5 For Exercise 15.3 we would need δ approximately equal to 2.50, 2.80, and 3.25 for power of .70, .80, and .90, respectively.

$$\delta = \gamma\sqrt{N}$$

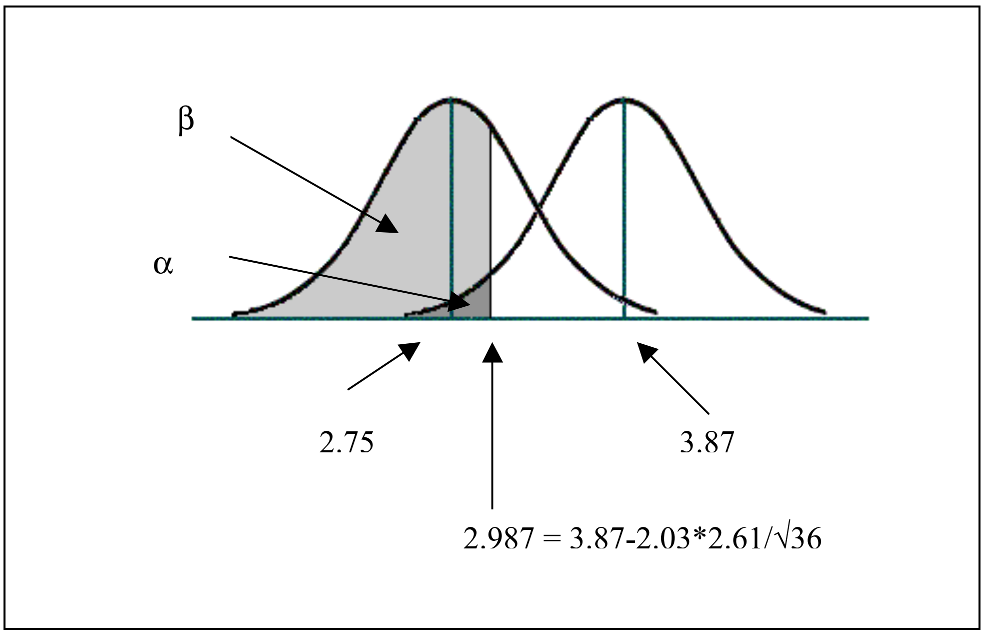
$$2.50 = .20\sqrt{N} \text{ therefore } N = \left(\frac{2.50}{.20}\right)^2 = 156.25$$

$$2.80 = .20\sqrt{N} \text{ therefore } N = \left(\frac{2.80}{.20}\right)^2 = 196$$

$$3.25 = .20\sqrt{N} \text{ therefore } N = \left(\frac{3.25}{.20}\right)^2 = 264.06$$

> Notice how quickly the required sample sizes increase, and how as *p* increases the *N* required increases faster and faster.

15.7 Diagram of Exercise 15.6:



15.9 Avoidance behavior in rabbits using a one-sample *t* test:

a) For power = .50 we need δ = 1.95.

$$\gamma = \frac{\mu_1 - \mu_0}{\sigma} = \frac{5.8 - 4.8}{2} = \frac{1.0}{2} = .50$$

$$\delta = \gamma\sqrt{N}$$

$$1.95 = .5\sqrt{N} \text{ therefore } N = \left(\frac{1.95}{.50}\right)^2 = 15.21$$

b) For power = .80 we need δ = 2.80.

$$\delta = \gamma\sqrt{N}$$

$$2.8 = .5\sqrt{N} \text{ therefore } N = \left(\frac{2.8}{.50}\right)^2 = 31.36$$

Because subjects come in whole units, we would need 16 subjects for power = .50 and 32 subjects for power = .80

15.11 Avoidance behavior in rabbits with unequal sample sizes:

$$\gamma = .50$$

$$N = \overline{N}_h = \frac{2N_1N_2}{N_1 + N_2} = \frac{2(20)(15)}{20+15} = 17.14$$

$$\delta = \gamma\sqrt{N/2} = .5\sqrt{\frac{17.14}{2}} = 1.46$$

With δ = 1.46, power = .31

15.13 Cognitive development of LBW and normal babies at 1 year—modified data:

a) Power calculations

$$\gamma = \frac{\mu_1 - \mu_0}{\sigma} = \frac{25-28}{8} = -.375$$

$$\delta = \gamma\sqrt{\frac{N}{2}} = -.375\sqrt{\frac{20}{2}} = -1.19$$

With δ = -1.19, power = .22

b) *t* test:

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{s_p^2}{N_1} + \frac{s_p^2}{N_2}}} = \frac{25-28}{\sqrt{\frac{64}{20} + \frac{64}{20}}} = -1.19$$

[$t_{.05}(38) = \pm 2.205$] Do not reject the null hypothesis.

c) The *t* is numerically equal to δ, although *t* is calculated from statistics and δ is calculated from parameters. In other words, δ is equal to the *t* we would get if the data came out with statistics equal to the parameters, 15.15 The significant t with the smaller N is more impressive, because that test had less power than the other, so the underlying difference is probably greater.

The fact that a significant difference with a small N is more impressive should not lead the student to conclude that small sample sizes are to be preferred.

15.17 Social awareness of ex-delinquents—which subject pool would be better to use?

$\bar{X}_{Normal} = 38 \quad N = 50$

$\bar{X}_{College} = 35 \quad N = 100 \qquad \bar{X}_{Dropout} = 30 \quad N = 25$

$$\gamma = \frac{38-35}{\sigma} \qquad \gamma = \frac{38-30}{\sigma}$$

$$\bar{N}_h = 66.67 \qquad \bar{N}_h = 33.33$$

$$\delta = \frac{3}{\sigma}\sqrt{\frac{66.67}{2}} \qquad \delta = \frac{8}{\sigma}\sqrt{\frac{33.33}{2}}$$

$$= \frac{17.32}{\sigma} \qquad = \frac{32.66}{\sigma}$$

Assuming equal standard deviations, the H. S. dropout group of 25 would result in a higher value of δ, and therefore higher power.

15.19 Total Sample Sizes Required for Power = .60, $\alpha = .05$, Two-Tailed ($\delta = 2.20$)

Effect Size	γ	**One-Sample *t***	**Two-Sample *t*** ***(per group)***	**Two-Sample *t*** ***(overall)***
Small	0.20	121	242	484
Medium	0.50	20	39	78
Large	0.80	8	16	32

15.21 When can power = β?

The mean under H_1 should fall at the critical value under H_0. The question implies a one-tailed test. Thus the mean is 1.645 standard errors above μ_0, which is 100.

$\mu = 100 + 1.645\sigma_X$

$= 100 + 1.645(15)/\sqrt{25}$

$= 104.935$

When $\mu = 104.935$, power would equal β.

15.23 The power of the comparison of TATs of parents of schizophrenic and normal subjects.

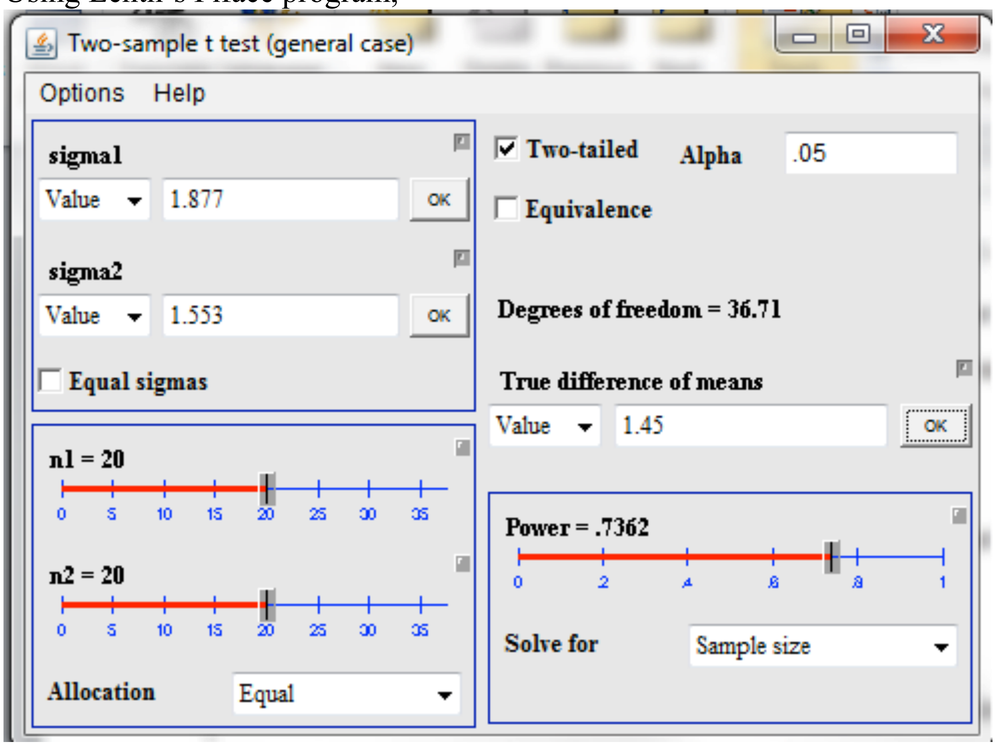
$$s_p^2 = \frac{3.523 + 2.412}{2} = 2.968; \quad s_p = \sqrt{2.968} = 1.723$$

$$\gamma = \frac{\mu_1 - \mu_2}{\sigma} = \frac{3.55 - 2.10}{1.723} = \frac{1.45}{1.723} = 0.842$$

$$\delta = \gamma\sqrt{\frac{N}{2}} = 0.842\sqrt{\frac{20}{2}} = 2.66$$

Power = .75

Using Lenth's Piface program,



15.25 I will use *R* to solve Exercise 15.3, but the rest of the solutions can follow using modifications of this same code.

```
### The following is the important information from the help file
for pwr.t.test.
#pwr.t.test(n = NULL, d = NULL, sig.level = 0.05, power = NULL,
          type = c("two.sample", "one.sample", "paired"), alternative =
c("two.sided",
               "less","greater"))
#Arguments
#n          Number of observations (per sample)
#d             Effect size
```

```
#sig.level     Significance level (Type I error probability)
#power              Power of test (1 minus Type II error probability)
#type       Type of t test : one- two- or paired-samples
#alternative a character string specifying the alternative hypothesis,
must be one of "two.sided" (default), "greater" or "less"

# First I will calculate the effect size for Ex15.3
mu1 = 4.39
mu0 <- 3.87
sp <- 2.61
d <- (mu1 - mu0)/sp
pwr.t.test(n = 36, d = d, sig.level = .05, type = "one.sample")


     One-sample t test power calculation
              n = 36
              d = 0.1992337
      sig.level = 0.05
          power = 0.2135633
    alternative = two.sided
```

There is a lot of software available over the Internet to do power analyses. A very complete listing and review of sources can be found at http://www.zoology.ubc.ca/~krebs/power.html .